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# Computational Modelling of Hand-Eye Coordination [and Discussion]

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# Computational modelling of hand–eye coordination

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## SUMMARY

We are developing a visually guided robot arm that is able to grasp and transport objects across an obstacle-strewn environment. Recent work has shown how analysis of moving image contours can provide estimates of the shape of curved surfaces. This forms the basis of the robot's capabilities which we are seeking to elaborate in three respects. First a computational model – the 'dynamic contour' – is being developed for real-time visual tracking, based on simulated Lagrangian Dynamics. Secondly we are developing a two-and-half-dimensional system for incremental, active exploration of free-space. Thirdly a computational theory of the visual planning of two-fingered grasps has been developed which can be coupled to the 'dynamic contour' model. This paper concentrates on the third of these topics. A qualitative theory for classification of grasps is described and demonstrated. Optimal finger positions are obtained from the mutual intersection of the local symmetry and anti-symmetry sets, and of a third set, the critical set of the grasp map. These three sets themselves form boundaries in a natural partition of the set of all grasps (the configuration space). The partition corresponds to a classification of possible grasps according to their stability properties.

## 1. INTRODUCTION

This paper outlines a model for planning grasping actions. This is done in the context of a robot system driven by 'active vision' (Aloimonos *et al.* 1987; Kass *et al.* 1987; Ballard 1991). We are attempting to build a system that links perception with action in real time. The past ten years has seen a gradual movement away from the Marrian view of vision (Marr 1982) as a static, information-gathering process. The earlier view of Gibson (1979) of vision directly linked to action is attractive, particularly for certain low-level perceptual tasks such as catching, and moving around obstacles. This partial retrenchment is encouraged by practical advances in computing, especially in parallel computing. As the power of parallel processors has increased, the demands on them by vision processes, linked more and more directly to action, has decreased. The result has been the emergence of practical visually driven robots (Faugeras & Hebert 1986; Brady *et al.* 1987; Dickmanns & Graefe 1988; Pichon *et al.* 1989; Rimey & Brown 1992).

We are currently building visually guided grasping expertise into a robot which can reach around obstacles. A CCD camera is mounted on the wrist of an ADEPT robot and a computer continually monitors visual signals. Deliberate, exploratory motions are coordinated with visual computation so as to explore the shape of three-dimensional, curved obstacles (Giblin & Weiss 1987; Blake & Cipolla 1990; Cipolla & Blake 1990). The visual computation itself is based on dynamic contours (Kass *et al.* 1987; Curwen & Blake 1992) which are strongly focused onto obstacle silhouettes and hence efficient. They are computed in parallel. The combination of focused attention and parallelism leads to real-time performance. This in turn allows integration of perception with control systems for action (figure 1). Coordinated, sequenced perception and action drives the robot to reach around obstacles towards a goal (Blake *et al.* 1991; Blake *et al.* 1992), as in figure 2. Currently, the robot is able to reach around obstacles and retrieve objects. Equipped with a suction gripper it is able to grasp only relatively flat objects. For instance, it can locate a hidden box and remove its lid. For more flexible grasping it would be desirable to use a gripper with fingers. Complex, anthropomorphic hands have been built (Salisbury & Roth 1983) but, for control purposes, the simplest gripper is a two-fingered one. The fingers may open and close either with parallel motion or with hinged motion. They may either be plates or thin cylinders. Of course the human hand has a wide repertoire of grasps (Elliot & Connelly 1984). The assumptions of our model are probably closest to the index roll, a grip between thumb and index finger.

In this paper we outline a theory for relating the control of a two-fingered gripper to the information available directly from visual contours. We build on the notable work of Faverjon & Ponce (1991), extending it to obtain qualitative rules for equilibrium and stability of grasp based on contour shape. The result is a computational procedure for obtaining seed grasps, locally optimal finger positions which seem to be intuitive grasps in many cases. The classification of seed grasps is based on an elementary use of differential analysis, of the kind used by Bruce & Giblin in their taxonomy of symmetry sets (Bruce & Giblin

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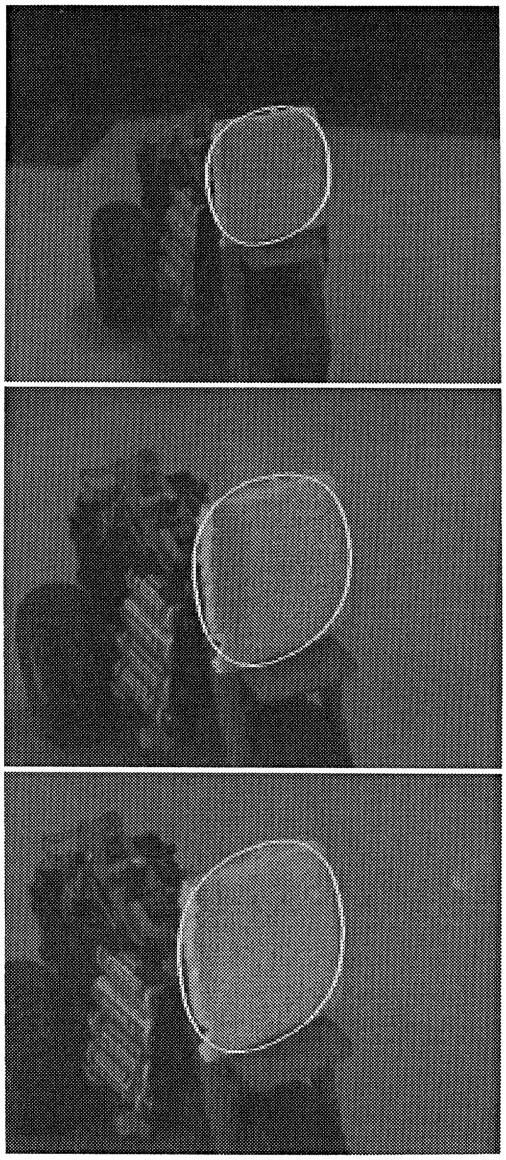

Figure 1. A 'dynamic contour' visual tracker follows the motion of a model vehicle. The camera is mounted on the wrist of an ADEPT robot which rotates the camera to maintain the contour in the centre of the image. The contour, a computed B-spline, is shown in white, overlaid on the images. (Figure by courtesy of R. Curwen.)

1984). Indeed the classification of grasps turns out to be closely coupled to symmetry properties.

Results of a practical computer algorithm based on this analysis of seed grasps indicates their value as qualitative predictors of grasp stability. Figures 3 and 4 show object outlines procured from our active contour tracker. The robot directs the camera's gaze towards the object to be grasped and the dynamic contour is allowed to lock onto the outline. Then the qualitative analysis of grasp is applied to the contour and distinguished grasp points – the seed grasps – are marked. They can be classified type 1 and type 2. Type 1 seeds remain in equilibrium as the coefficient of friction tends to zero whereas type 2 require some minimum degree of friction for equilibrium. In the case of type 1 points, a measure of stability can also be computed. A reasonable grasping strategy is to try the most stable grasps first. The remainder of the paper explains how a theory of two-fingered grasps can be constructed which leads to the computational results like those just shown.

## 2. EQUILIBRIUM OF GRASPS

We assume, in this paper that a thin, planar object bounded by a closed curve is to be grasped by a two-fingered gripper. Only the two-dimensional problem will be considered in which the object rests on a plane. It is to be gripped by fingers that move only in that plane. Gravitational effects are not considered. The question is simply whether, when gripped, the object remains static. Imagine grasping a coin resting on a highly polished table: how can the coin be gripped in two fingers without it shooting across the table?

In order to study grasps that are in equilibrium in this sense, a mathematical condition for equilibrium is needed. It is standard to use the 'force-closure' property (Latombe 1991) which is a sufficient condition for equilibrium. A set of contact points on a body satisfies the closure condition if any given force and couple on the body can be achieved by some set of positive normal forces at the contacts. In this paper we consider conditions for force-closure with frictional fingers. In the case of a two-dimensional polyhedral

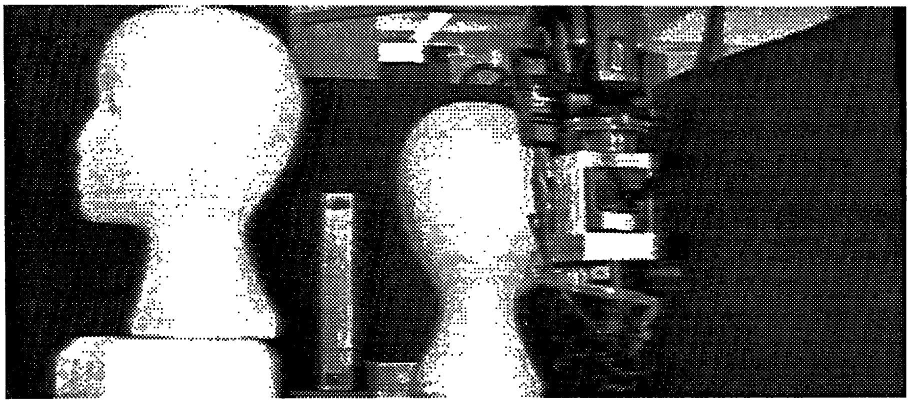

Figure 2. The robot directs the camera's gaze towards one of two obstacles, attempting to plan a path between them. The goal object is marked by the vertical white column in the distance. (Figure by courtesy of A. Zisserman.)

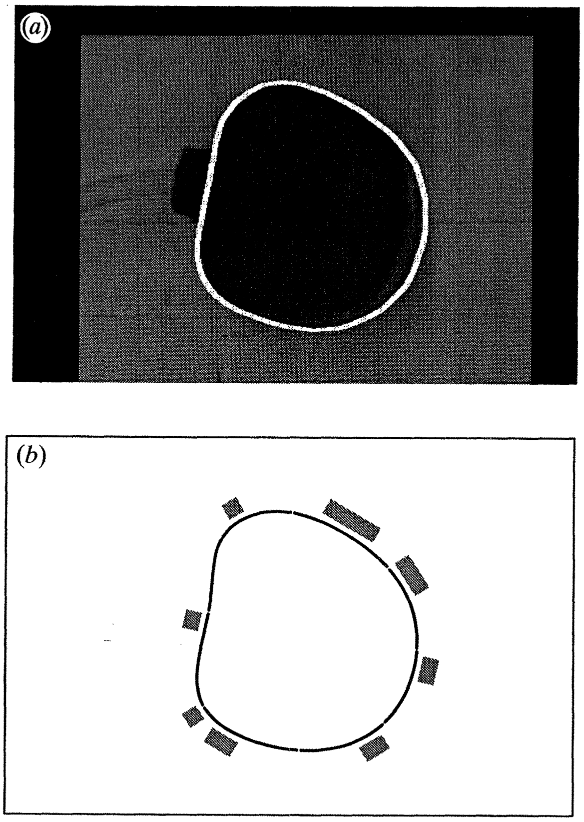


Figure 3. (*a*) An electrical plug, as seen by an ADEPT robot's wrist-mounted camera. Its outline is captured by a 'dynamic contour' tracker. Once captured, the contour is frozen and analysed for grip. (*b*) Seed grips shown here are all of 'type 1', grips that remain in equilibrium as the coefficient of friction approaches zero. They are parallel grips, in which the two fingers (depicted by rectangles) are placed at opposing points on the contour. Four such grips are shown here. The length of each rectangle depicts a measure of stability for one finger in a two-fingered grasp.

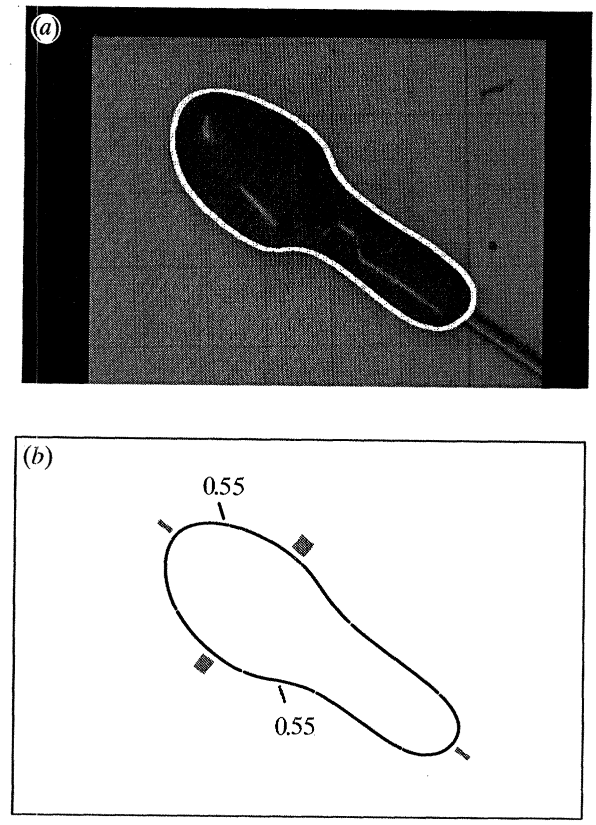


Figure 4. (*a*) A screwdriver, as seen by the robot's camera, with a 'dynamic contour' locked onto the outline of its handle. (*b*) There are four seed grasps of type 1 (see figure 3), the most stable of which are the natural grasps across the handle. The other, much less stable but still feasible, is longitudinal.

body, Markenscoff *et al.* (1990) showed that three such fingers are always sufficient for force-closure. This assumes that vertices are to be considered ungraspable: an appropriate assumption for thin fingers but unduly conservative when the fingers are plates, as in the parallel-jaw gripper or when grasping a smooth curve.

Given that we are using B-splines to represent visual contours it is more natural to consider two-dimensional bodies as being bounded by a smooth contour, rather than as a polyhedron. Then, it transpires, every contour has at least one force-closure grasp with just two contact points. Following Nguyen (1988) and Faverjon & Ponce (1991), the force-closure of a two-fingered grasp can be established by a simple geometric test, expressed in the notation of figure 5. The contour itself is denoted $\boldsymbol{r}(s)$ where $s$ is a parameter which will usually be taken to be true arc-length. Grasp points are at $s=s_1,s_2$. Friction angles $\alpha_1,\alpha_2$ are defined at the two points. Given a coefficient of friction $\mu$, the test is passed whenever

$$|\tan\alpha_1|<\mu \quad \text{and} \quad |\tan\alpha_2|<\mu. \tag{1}$$

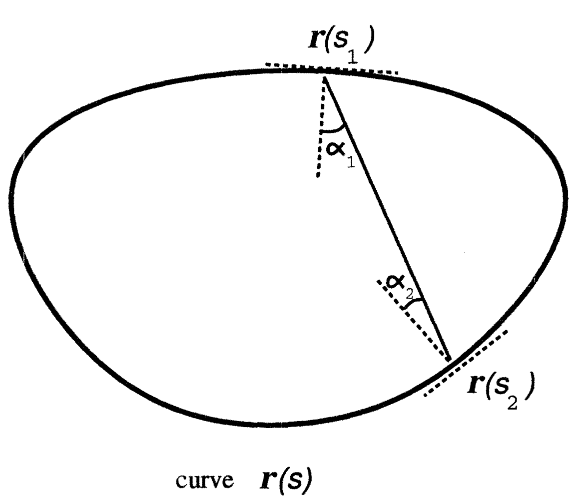


Figure 5. We consider a two-fingered grip of a smooth contour (usually closed) $\boldsymbol{r}(s)$, placing the fingers at $s=s_1,s_2$ respectively. The line joining the two fingers makes angles $\alpha_1,\alpha_2$ with the normals at $s=s_1,s_2$ respectively.

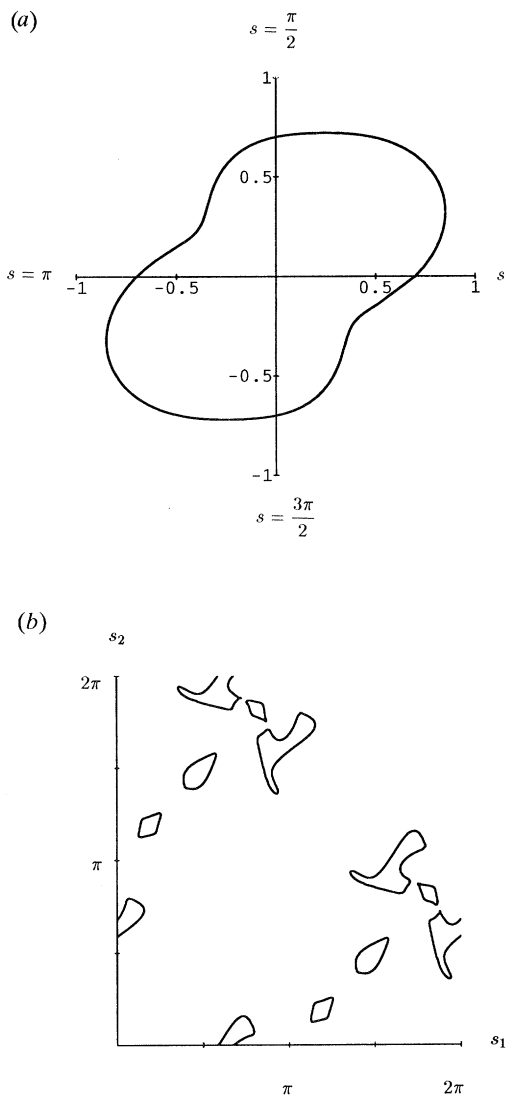


Figure 6. Two-fingered equilibrium grasps of the contour $\boldsymbol{r}(s)$ in (*a*) can be represented as regions in C-space. This is shown in (*b*) for the case that the coefficient of friction is $\mu = \frac{1}{3}$ Each axis represents the position $s_i$, $i = 1,2$ of the $i$th finger along the closed curve $\boldsymbol{r}(s)$. (In this case $s$ is not arc-length but polar angle relative to the origin.) Note that equilibrium grasps are organised into just a few discrete regions.

For a given coefficient of friction $\mu$, these conditions define a region in 'C-space' (configuration space for a two-fingered grasp of the curve $\boldsymbol{r}(s)$), as in figure 6. Grasps within a region are in equilibrium at the given $\mu$. Each region is bounded by the four curves

$$\tan \alpha_i = \pm \mu, \qquad i = 1, 2. \tag{2}$$

Faverjon & Ponce (1991) give an efficient procedure for computing them. What is clear from the figure is that there are just a few distinct regions, each corresponding to natural grips on the contour. Note the symmetry of the diagram about the $s_1 = s_2$ line arising because force closure is invariant to a direct swap of finger positions. The diagram is also periodic over $2\pi$ both in $s_1$ and $s_2$ because the contour $\boldsymbol{r}(s)$ is closed. Thus the diagram is topologically toroidal with

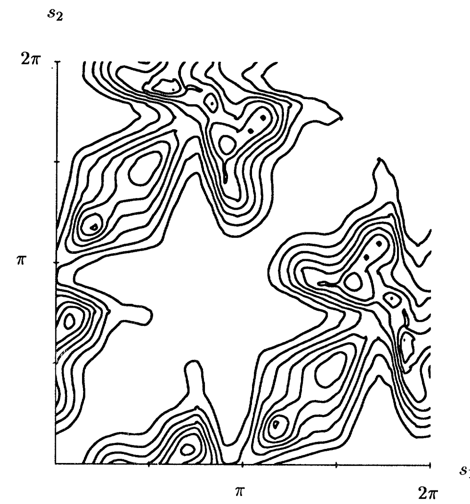


Figure 7. The full structure of C-space is given by this C-space diagram. It reproduces regions like those in figure 6*b* for all levels of friction. It is a contour plot of the function $S(s_1,s_2)$ (see text).

the upper pair of edges identified and similarly the left and right edges. Allowing for periodicity and the symmetry there are in fact only five distinct regions in the diagram and hence there are five qualitatively distinct grasps.

A weakness of the model so far is the requirement to specify a level of friction $\mu$. It is attractive to replace this requirement by the much weaker assumption that the two fingers merely have the same coefficient of friction. In that case regions in C-space must be replaced by the 'C-space diagram': a contour map showing the regions as a function of $\mu$. Such a diagram is shown in figure 7, a contour plot of the 'friction function'

$$S(s_1,s_2) = \max(|\tan \alpha_1|, |\tan \alpha_2|), \tag{3}$$

so that grasp regions for a given $\mu$ are simply those contained in the set

$$\{s_1,s_2\colon\ S(s_1,s_2) < \mu\}.$$

The diagram contains, in principle, complete information about the structure of C-space but, given its complexity, it is compelling to seek a concise set of qualitative features conveying the essential structure. A natural candidate is the set of minima of the diagram which represent locally optimal grasps: termed seed grasps. Such a grasp is optimal in the sense that any perturbation of finger positions causes an increase in $S(s_1,s_2)$. Hence any perturbation of a seed grasp requires a higher level of friction to be in equilibrium. It is clear from the C-space diagram that there can only be a few seed grasps in the case illustrated. Note that each of the five distinct regions in figure 6*b* is accounted for by minima in the C-space map.

It seems that the seed grasps may afford a succinct representation of the structure of grasp C-space. A reasonable grasping strategy would be to try to grasp at or near a seed grasp. The next two sections explain

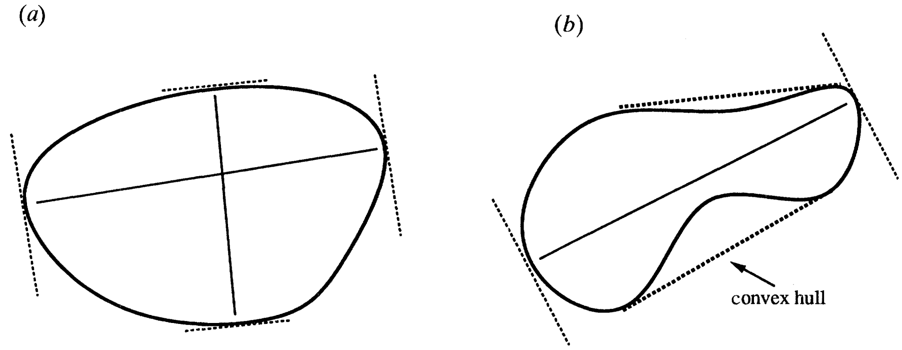


Figure 8. (*a*) Every convex contour has at least two type 1 (zero-friction) grasps. (*b*) A non-convex contour has at least one type 1 grasp located, moreover, on a part of the convex hull which is in contact with the curve itself. Hence this grasp is guaranteed to be accessible to a parallel jaw gripper.

how seed grasps can be efficiently located and their relationship to the geometry of the contour $\boldsymbol{r}(s)$.

## 3. PRIMARY GRASP STRUCTURE

Seed grasps have been defined as local minima of the friction function $S$ in equation (3) and could occur as either of two types.

Type 1: a grasp for which $\alpha_1 = \alpha_2 = 0$ so that, from equation (3), $S=0$. Since generally $S \geqslant 0$, the case $S=0$ is a minimum, representing a grasp that is in equilibrium right down to zero friction.

Type 2: a grasp for which $S$ is minimal but $S \neq 0$. Such a grasp is locally optimal (any nearby perturbation of it increases the friction needed to guarantee stability) but is not in equilibrium down to zero friction.

A natural question concerns the existence of type 1 and type 2 grasps. Since $S$ is positive, and defined on a compact set, it must achieve a minimum in C-space, at least once. So either type 1 seeds or type 2 seeds must exist but not necessarily both.

In fact it is easy to see that any smooth contour $\boldsymbol{r}(s)$ must have at least one type 1 seed and this is illustrated in figure 8. First, in the case of a convex contour, imagine applying a caliper to the contour to measure its diameter in a particular direction. Then, with the caliper applied, rotate the object so that the caliper opens and closes as the object rotates. There must be at least one position of maximum diameter and one minimum. Each is a type 1 seed grasp because the line joining the fingers is perpendicular to the curve tangents at each finger (figure 8*a*). Hence every convex contour has at least two type 1 seeds. Indeed, generically there will be an even number of type 1 seeds on a convex contour, corresponding to the maxima and minima of caliper diameter.

In the non-convex case (figure 8*b*) we can construct the convex hull of the contour which, being convex, must have at least two type 1 seeds. For the one corresponding to the maximum diameter, both fingers must also lie on the original contour (figure 8), hence the contour has at least one type 1 seed. Moreover that seed is accessible in the sense that it lies on the contour's convex hull so that it can be reached not only by a thin fingered gripper but also by a parallel jaw gripper whose fingers are plate-shaped.

We have seen that at least one seed grasp must exist on every smooth contour but also that there must exist at least one type 1 seed. Is it then possible that all seeds are in fact of type 1 and that the type 2 seeds never occur? This possibility is attractive because it would mean that the C-space map would be entirely characterised by grasp properties at zero friction. Increasing friction would increase the size of C-space regions (figure 6*b*) but no new regions would arise. This is somewhat reminiscent of the 'structure-preservation' properties of Witkin's 'scale-space diagram' (Witkin 1983; Yuille & Poggio 1984) in which all structure in a signal is present at fine scale; no new structure can arise as the scale on which the signal is viewed coarsens.

It turns out that the 'structure-preservation' hypothesis for grasp space does not hold. New regions in C-space can be created as friction increases. In fact the C-space map of figure 7 contains a counterexample to the hypothesis. The minimum of $S(s_1,s_2)$ at $(s_1,s_2) = (\frac{\pi}{2}, \frac{3\pi}{2})$ is at a non-zero value of $S$, representing a seed grasp requiring greater than zero friction for equilibrium. It is a grasp along the vertical axis on the contour itself (figure 6*a*). It can plainly be seen that the vertical axis is not orthogonal to the contour at either intersection with the contour and hence fingers placed at those intersections would require greater-than-zero friction. Yet (and this, admittedly, is less obvious by inspection) that grasp is locally optimal. Any perturbation of the fingers produces a grasp requiring even more friction for equilibrium. Hence it is a type 2 seed.

A construction for another counterexample is given in figure 9 and gives some insight into why type 2 seeds occur. The contour in figure 9*a* is simply a piece of an equilateral triangle; the minimum friction for any grasp to be in equilibrium is clearly $\tan \alpha$, where $\alpha$ is the half-angle at the apex. Now this is not quite what is required because there is no isolated minimum-friction grasp. Rather, minimal friction grasps form a line in C-space, corresponding to the set of all mirror-symmetrical grasps of the contour. This is non-generic. However, a perturbation of the contours can cause an isolated minimum friction grasp to occur

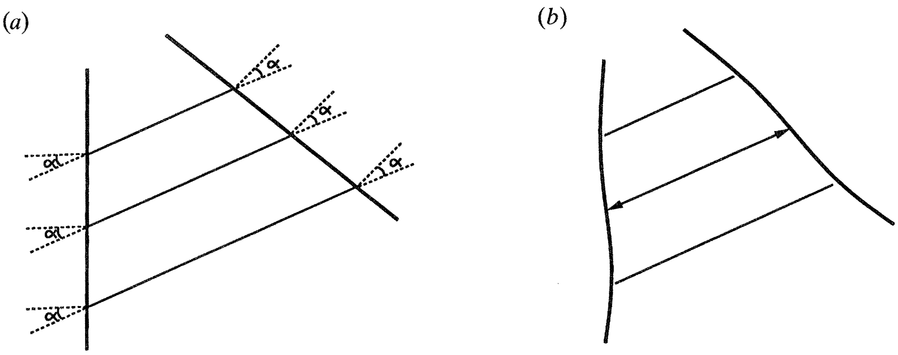


Figure 9. This counterexample shows that structure is not preserved in grasp-space. (*a*) Minimum friction ($\mu = |\tan\alpha|$) grasps on this pair of lines form a continuum as shown. (*b*) A small perturbation introduces a unique minimum friction grasp, shown arrowed. However, the coefficient of friction $\mu$ at that minimum is greater than zero. Hence a region appears, at the corresponding point in grasp-space, at a coefficient of friction that is strictly greater than zero.

as in figure 9*b*. This is a type 2 seed grasp. Note that this counterexample is for the case of a non-parallel grasp – one that would be inaccessible to a parallel-jaw gripper – unlike the previous counterexample.

## 4. FURTHER STRUCTURE: SYMMETRY SETS AND CRITICAL SETS

The qualitative structure of the C-space map can be represented in terms of the type 1 and type 2 seed grasps. The type 2 grasps occur in one of two forms: anti-symmetric (2a) and symmetric (2s). Figure 10 shows that each form is a configuration of local symmetry, as determined by the matching of angles $\alpha_1,\alpha_2$. The symmetric case (type 2s) is a 'local symmetry' in the sense defined by Brady & Asada (1984), and formalized as the 'symmetry set' by Bruce & Giblin (1984). The anti-symmetric case (type 2a) is similar to the local rotational symmetry explored by Fleck (1986). Some effort has been applied to the problem of depicting the symmetry set as a set of image curves, overlaid on the contour $\boldsymbol{r}(s)$. Brady & Asada (1984), Bruce & Giblin (1984) and Leyton (1988) each propose somewhat different solutions. That debate is irrelevant here however as the symmetry set is to be viewed in C-space where the operational semantics of grasp are naturally represented, rather than pictorially. Then there is no latitude for alternative definitions. A symmetry (anti-symmetry) point in C-space is simply a point $(s_1,s_2)$ for which $\alpha_1 = \alpha_2 (\alpha_1 = -\alpha_2)$. This is shown for the case of an ellipse in figure 11*a*,*b*. A further simplification is that symmetry sets in C-space are generically smooth whereas when projected onto image-space they have special points: cusps, free ends and triple-crossings (Bruce & Giblin 1984).

Seed grasps are points in C-space; the symmetry sets are lines. Now type 1 seeds are simply the intersections of the symmetry set with the anti-symmetry set. This is clear since, at such an intersection, both $\alpha_1 = -\alpha_2$ (anti-symmetry) and $\alpha_1 = \alpha_2$ (symmetry) so that $\alpha_1 = \alpha_2 = 0$.

Type 2 seeds lie on the respective symmetry sets but are distinguished points on those sets. We require a further condition to localize them. It can be shown that type 2a and 2s seeds are points of intersection of the anti-symmetry and symmetry sets respectively with a third set: the critical set of the grasp map. This

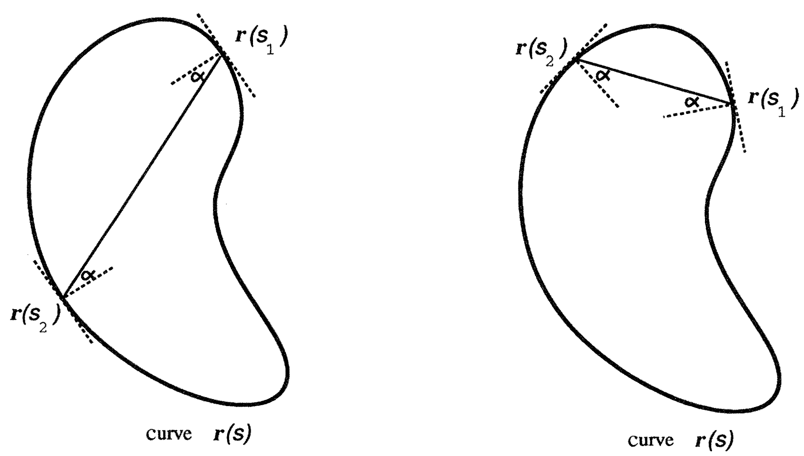


Figure 10. Local symmetries occur in one of two configurations. (*a*) Anti-symmetry (rotational symmetry) in which $\alpha_1 = -\alpha_2$ in figure 5. (*b*) Symmetry (mirror symmetry) in which $\alpha_1 = \alpha_2$.

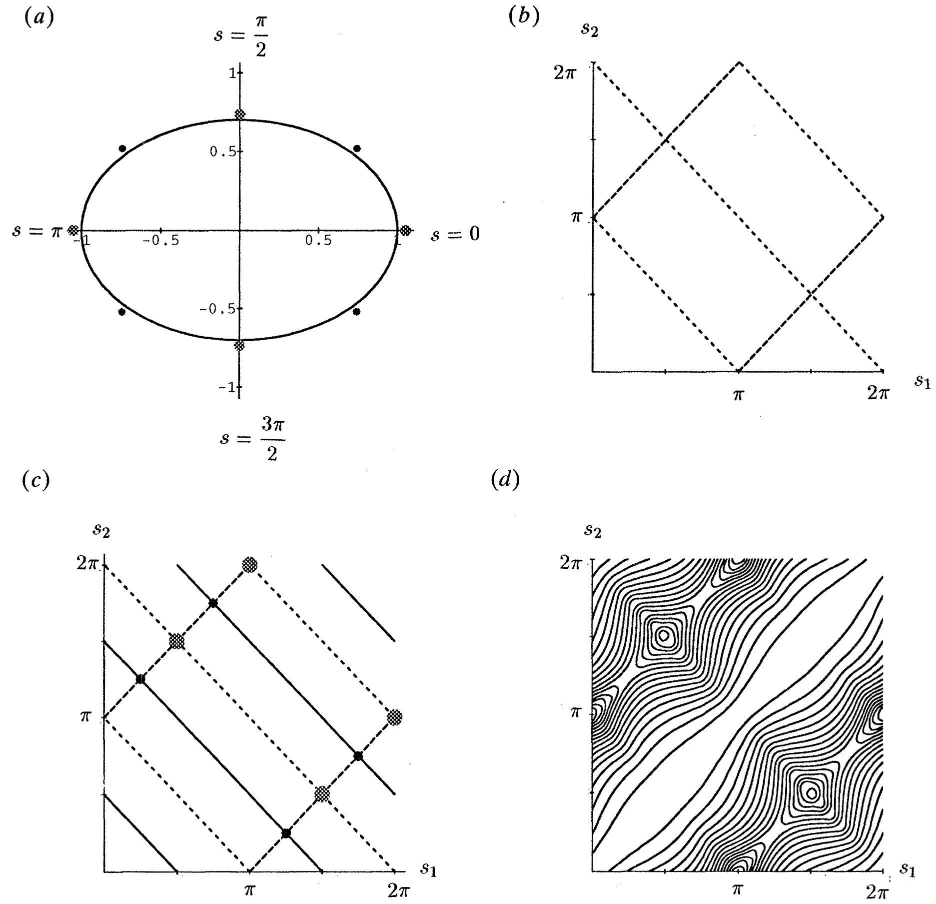


Figure 11. Symmetry sets for the ellipse in (*a*) are shown in (*b*): short dashes: symmetry set; long dashes: anti-symmetry set. In (*c*) the critical set (solid line) is also marked. Intersections of these sets correspond the extrema of the C-space map which is shown in (*d*). Large blobs in (*c*) are type 1 extrema, small blobs are type 2a. There are no type 2s extrema on this particular contour. The extrema are also illustrated as grasps on the contour $\boldsymbol{r}(s)$ in (*a*), involving pairs of opposing finger positions.

critical set can be defined algebraically be defining the 'grasp map'. Rather than do that here, an equivalent geometrical construction in terms of osculating circles is given in figure 12. The construction is a test that can be applied to any pair of points $s_1$,$s_2$ on a contour $\boldsymbol{r}(s)$. Generically, the set of points passing the test will form smooth curves in C-space.

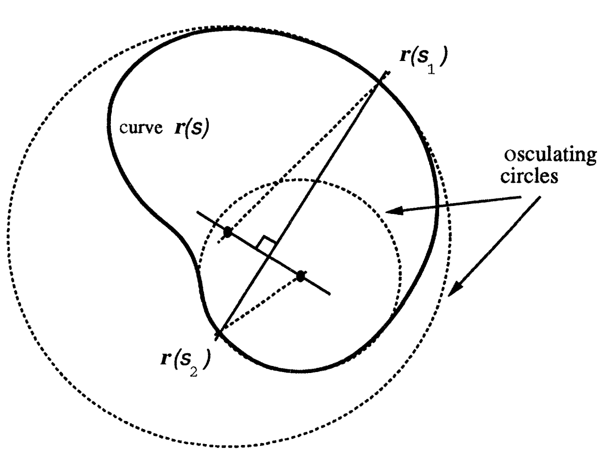


Figure 12. Critical points in grasp space occur under the geometrical condition illustrated; the line joining the finger positions $\boldsymbol{r}(s_1)$,$\boldsymbol{r}(s_2)$ is orthogonal to the line joining the centres of osculating circles. (The osculating circle at a point on a curve is defined to be tangent to the curve there and also to have the same radius of curvature.)

To summarize, all seed points are found as intersections of the three sets, as follows:

Type 1: Intersections of the symmetry set and the anti-symmetry set.

Type 2s: Intersections of the symmetry set and the critical set.

Type 2a: Intersections of the anti-symmetry set and the critical set.

This is illustrated for the elliptical case in figure 11*c*,*d*. Note however that, in the case of type 2 seeds, these intersection conditions are necessary but not sufficient. For type 2s, it is also necessary, generically, that the two fingers are on convex points of the contour. For type 2a, one must be at a convex point, the other at a concave point. That is sufficient to guarantee extremality of the friction function *S*, that is, either a saddle-point, maximum or minimum of friction. Minimal friction grasps are what were sought. Maximal friction grasps are also interesting being locally pessimal, indicating areas of C-space to avoid.

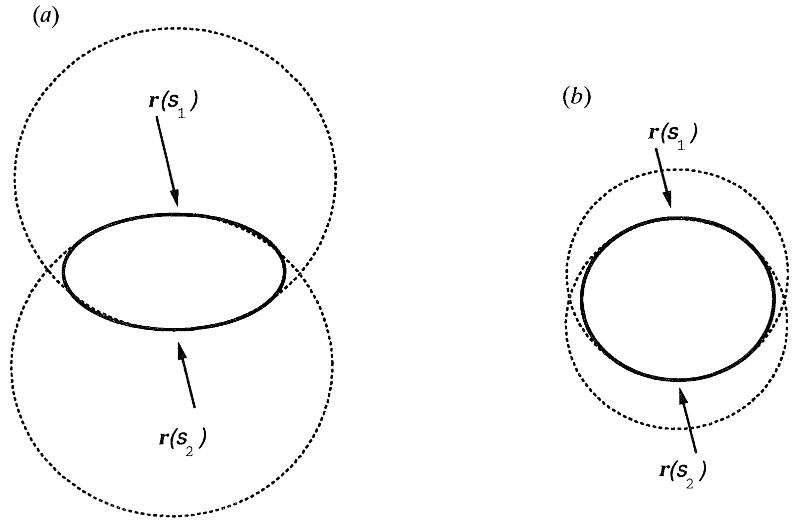


Figure 13. Stability of type 1 grasps can be measured in two different senses, static and dynamic. In either case the measure depends on the vector between the two finger positions and on the centres of the two osculating circles. The grasp (*a*) is stable in the static sense and (conditionally) in the dynamic sense. The grasp in (*b*) is dynamically less stable but statically more stable than the one in (*a*).

## 5. GRASP STABILITY

The qualitative description of C-space can be expanded by going beyond mere equilibrium (force-closure) to include stability. Given that a grasp is in equilibrium, a stable grasp is one for which equilibrium is maintained even when a perturbation has been applied. Various definitions of stability are useful: both for a thin-fingered gripper and for a parallel-jaw gripper, both dynamic and static stability. In all cases, stability properties are related to the critical set defined geometrically in figure 12.

Whereas the thin-fingered gripper has the two-dimensional $(s_1,s_2)$ C-space that we have been using so far, C-space for the parallel-jaw gripper must be a one-dimensional projection of $(s_1,s_2)$ space. It has only one degree of freedom, namely $\theta$ the orientation of the jaws relative to object coordinates. In fact, C-space for the parallel-jaw gripper is simply the anti-symmetry set itself (see, for example, figure 11*b*). General perturbations in C-space for the thin-fingered gripper are simply projected onto the antisymmetry set. Thus any grasp $(s_1,s_2)$ that is stable for a thin-fingered gripper must also be stable for a parallel-jaw gripper.

The distinction between dynamic and static stability is important. It depends on what type of perturbation is to be considered. Dynamic stability covers the case of perturbation of object position whereas static stability applies to a perturbation of the coefficient of friction.

Dynamic stability is more intuitive and this is probably because it is the more relevant sense for humans, whose eyes and hands are well separated. Any stable perturbations of the gripped object are acceptable because the eyes can monitor them continuously and keep track of the object's position. It is also the appropriate sense of stability for a 'frictionless' parallel-jaw gripper (Goldberg *et al.* 1991), one of whose jaws is free to slide longitudinally. A robot like the one pictured in figure 2, however, suffers from hypermetropia. It cannot see its own hand and in fact it cannot see anything closer than about one-tenth of the diameter of the workspace. Static stability is then more important. The part is visible before it is picked up. Once it is gripped it must not move relative to the fingers; any such motion would be unseen and the precise positioning of the part would be lost.

Statically, one perfectly stable grip is across the diameter of a circle. This corresponds to a point in two-fingered C-space at which the symmetry set, the anti-symmetry set and the critical set all meet. Stability then decreases as the centres of osculating circles move apart (figure 13). Separate stability measures can be computed for each finger and are displayed for the contours of real parts shown in figures 3 and 4. Finger positions for type 1 seed grasps are indicated there by rectangles whose length is a measure of stability of the finger. The rectangle indicates the safe range of movement of the finger when the coefficient of friction is 0.05 (perturbed, that is, from a value of 0.0).

Dynamic stability is defined by imagining the fingers to be connected by a spring and computing the potential energy $P(s_1,s_2)$ in the region of the equilibrium point. The equilibrium may be stable, conditionally stable or unstable according to whether $P$ is at a minimum, a saddle or a maximum. Again, it is the osculating circles that determine stability but in a way that is somewhat independent of the static notion of stability, as figure 13 shows.

Dynamic stability can be measured for any grasp, not just for a type 1 seed. In this more general case, transitions between the three types of stability occur exactly on the critical set in C-space. Hence C-space is partitioned by the critical set into regions according to the type of stability. In the case of figure 11*c*, regions of conditional stability (e.g. grasping the minor axis) alternate, in C-space, with regions of instability (e.g.

grasping the major axis). Furthermore, the symmetry and anti-symmetry sets also mark qualitative transitions of stability: namely, a transition of finger dominance. As a symmetry set is crossed the finger that is dominant, in the sense of requiring less friction for equilibrium, ceases to be dominant; the non-dominant finger becomes the dominant one.

## 6. COMMENTS AND CONCLUSIONS

Implementation of the theory so far covers the cases of type 1 and type 2a seed grasps. This is because a reasonably efficient algorithm is available for recovery of the anti-symmetry set from spline curves. Type 1 and type 2a seeds can be recovered by tracking along the anti-symmetry set in C-space, testing for intersections with the symmetry set and the critical set respectively. To recover type 2s seeds it would be necessary to track either along the symmetry set or the critical set. An approximate algorithm for tracking along the symmetry set of a B-spline curve has been devised by Rom & Medioni (1992). However, it is not yet clear whether such an approximation is close enough for our purpose.

An unexpected but pleasing outcome of this study has been the relationship between local symmetry and grasping. Previously, local symmetries have been seen as a descriptive tool, as a simplifying process for shape descriptors. Consideration of grasping however forces an operational description of shape. This has the effect both of confirming the descriptive value of local symmetry and modifying it. The most significant modification is to view symmetry in configuration space, rather than in image-space. This has the additional benefit of removing the mathematically fascinating but practically annoying problem of singularity that occurs in the image-space representation.

It is a matter for speculation whether human performance in visual grasping could be modelled using the ideas of seed grasps and grasp stability. Of course the theory here is two-dimensional. An extension to three-dimensions is yet to be fully worked out. However two-dimensional grasping is a significant subproblem that is convenient for psychophysical experimentation. There is psychophysical evidence (Barlow 1980) that the human vision system is highly efficient at recovering symmetry, even with exposure times as brief as 100 ms. An appealing hypothesis then is that our ability to judge stable grasp may be fed directly by the symmetry perception mechanism without recourse to cognitive mechanisms involving symbolic coding of shape.

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### *Discussion*

J. M. Brady (*Department of Engineering Science, University of Oxford, U.K.*). Suppose Dr Blake was to generalize his work to three fingers, say two fingers symmetrically opposing a 'thumb'. One approach might be to have a three-dimensional configuration space, but that would be rather complex. An alternative might be to replace the two actual fingers by a 'virtual finger' midway between them, then apply his current scheme and then refine it. Has he explored such schemes yet or analysed how three fingers might be useful?

A. Blake. The attractions of introducing a third finger are clear. In return for increased complexity both the static and the dynamic capabilities of the gripper would be enhanced.

The virtual finger suggestion relates to dynamic properties. Maintaining two virtual fingers whose mappings onto the physical fingers are constantly permuted (rather like Raibert's virtual leg (Raibert 1986)) opens up the possibility of dynamic behaviour. An object could be rotated by successive movements and exchanges of fingers, something like the radial roll in human grasping (Elliot & Connelly 1984). This is a most attractive idea, but probably not one on which the current model sheds much light.

Three fingers are also an advantage from the static point of view. In three-dimensions, force closure can be achieved with three frictional ('hard-contact' (Latombe 1991)) fingers. For instance two of the fingers may lie on an axis while the third finger acts as a brake for rotation about that axis. In two-dimensions, two fingers and an opposing thumb can be arranged to resist certain combinations of forces even in the absence of friction. A general analysis of this configuration requires a relatively cumbersome three-dimensional configuration space. If, however, it is assumed that two fingers are closely spaced, the two-dimensional configuration space of the current model would suffice. The two close fingers could be treated as a single finger plus a first-order perturbation, using the stability analysis for type 1 seed grasps.

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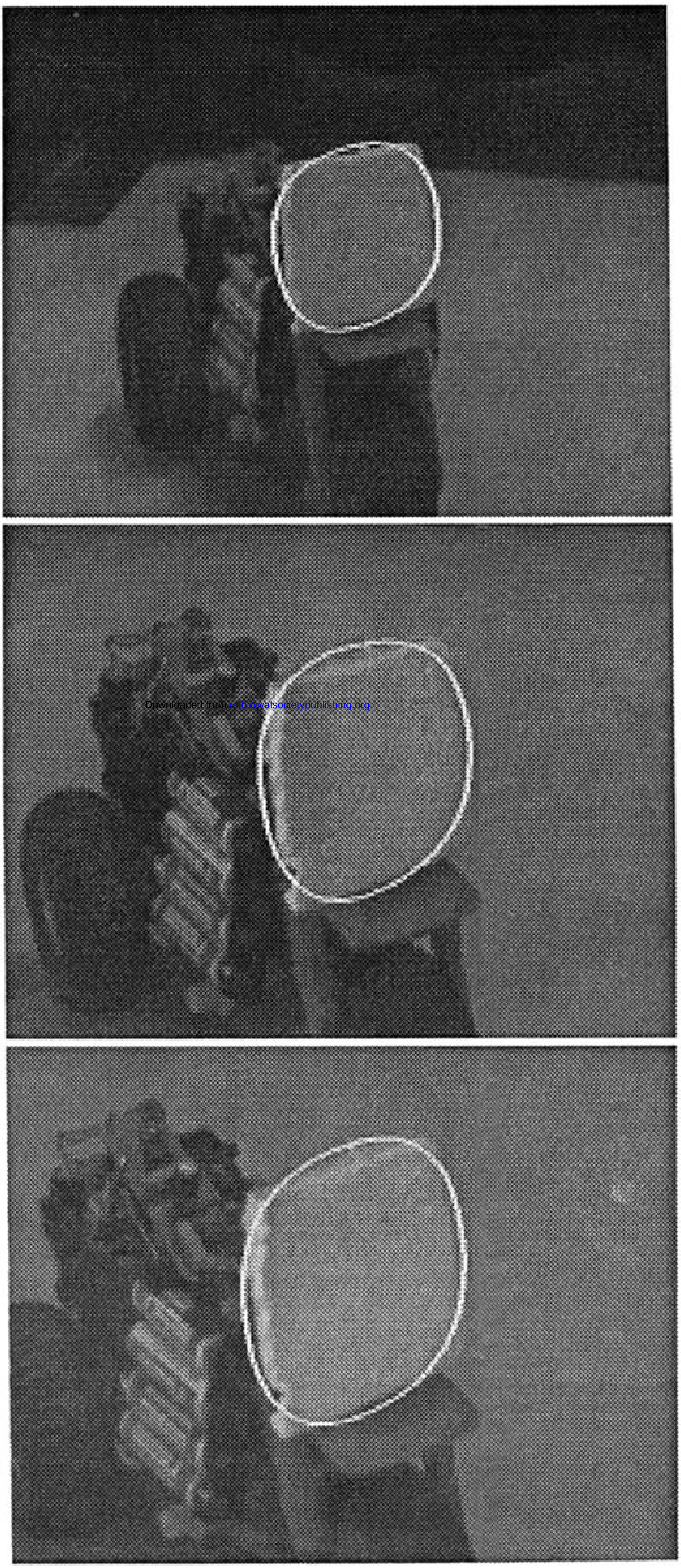
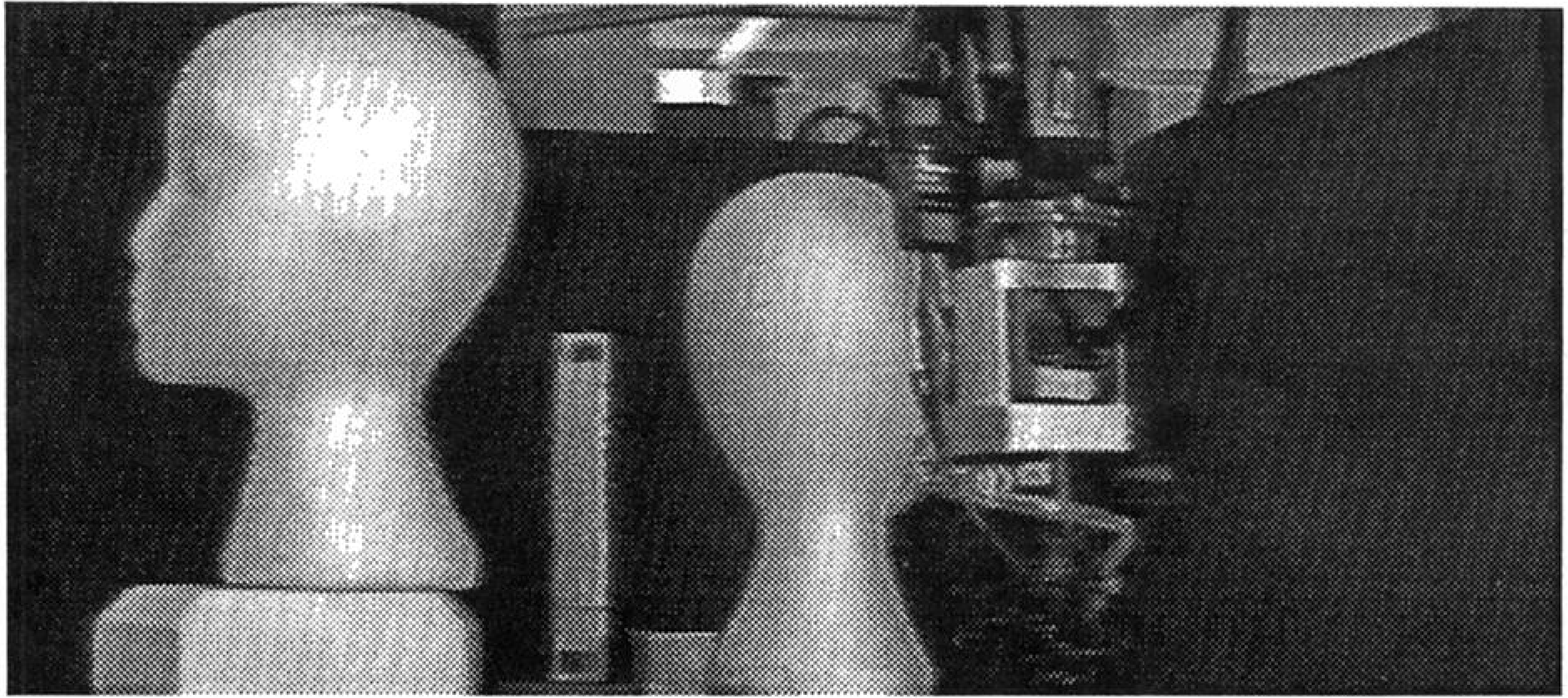
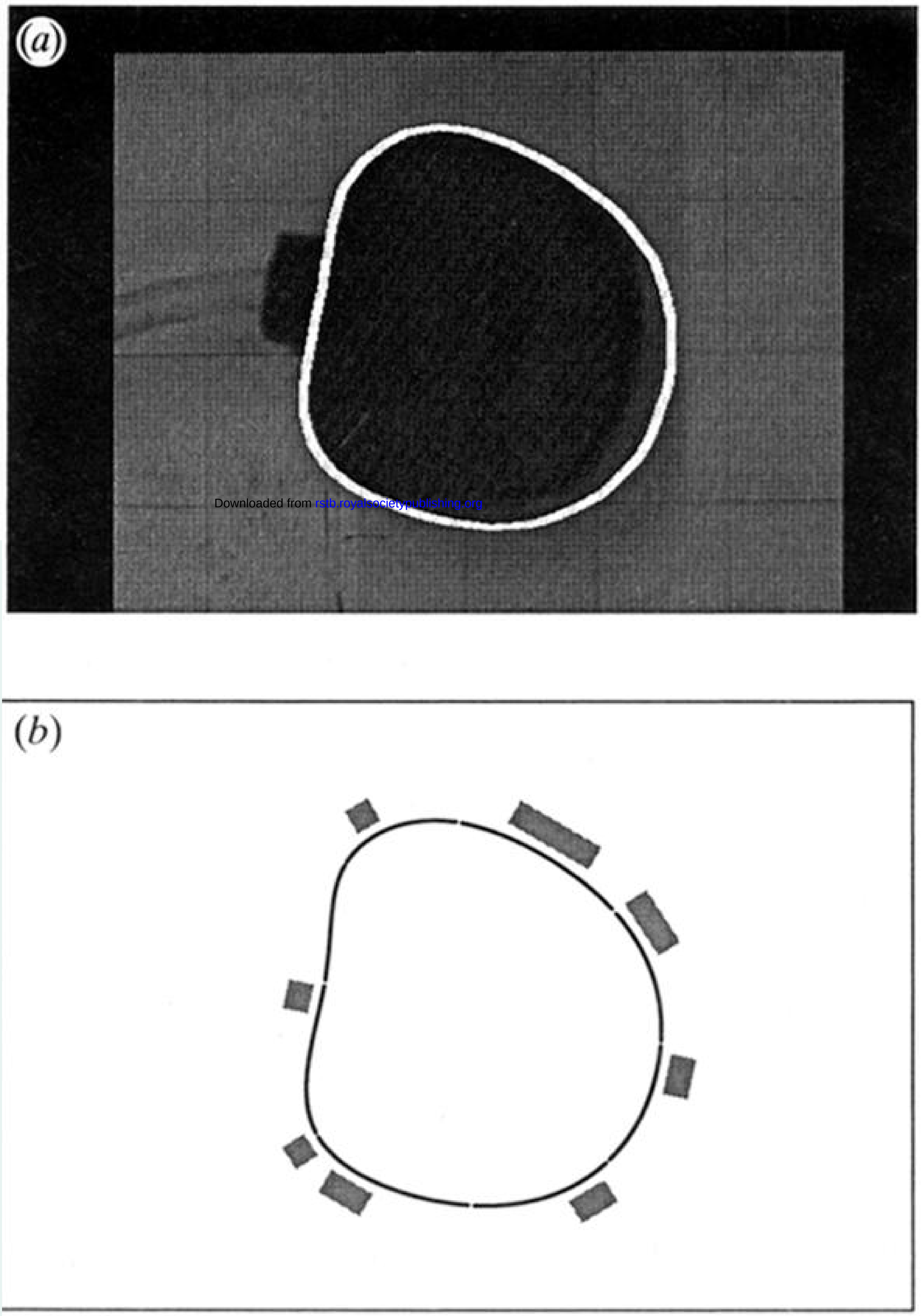




igure 1. A 'dynamic contour' visual tracker follows the otion of a model vehicle. The camera is mounted on the rist of an ADEPT robot which rotates the camera to aintain the contour in the centre of the image. The ontour, a computed B-spline, is shown in white, overlaid on e images. (Figure by courtesy of R. Curwen.)

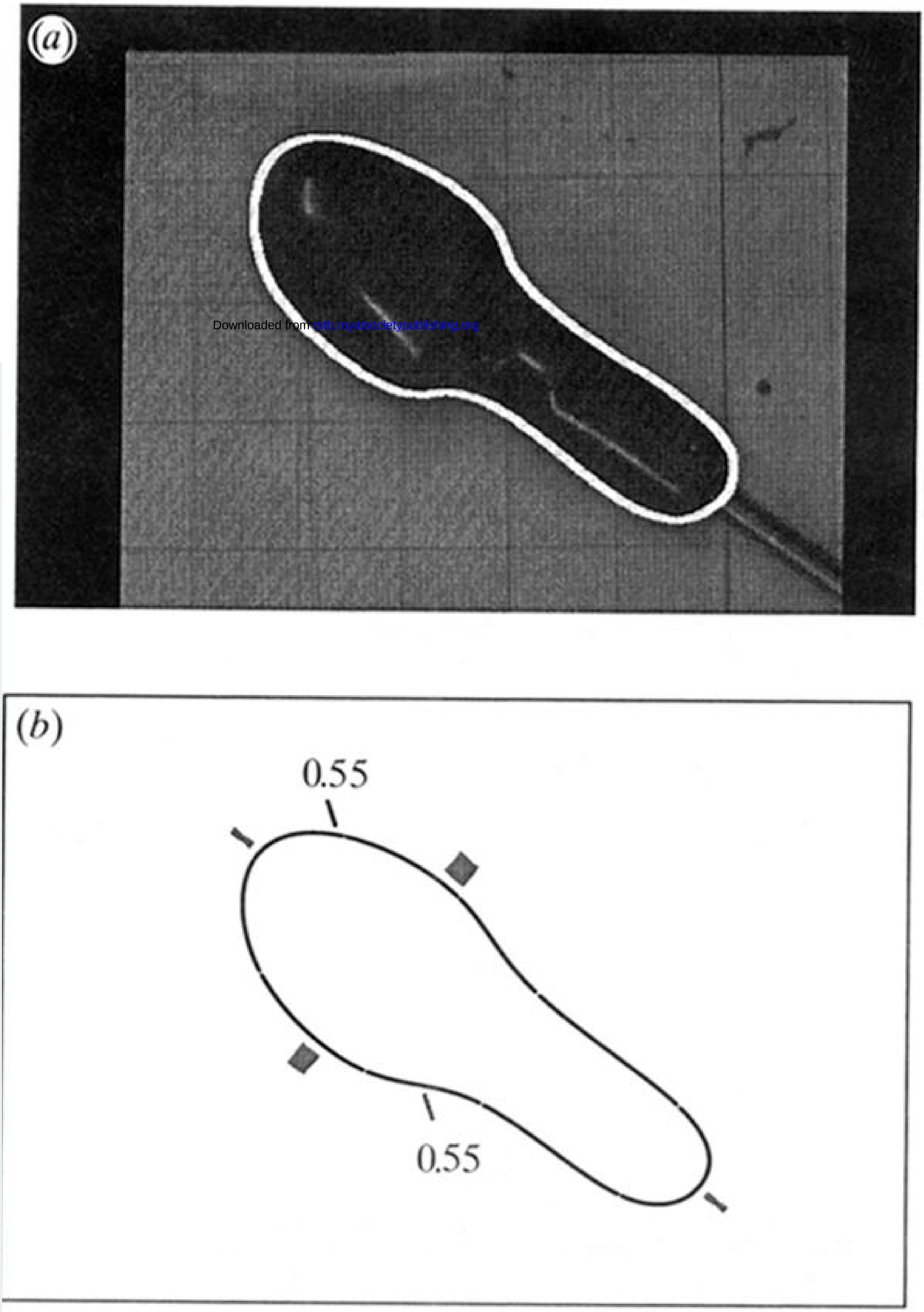


gure 2. The robot directs the camera's gaze towards one of two obstacles, attempting to plan a path between them. 'he goal object is marked by the vertical white column in the distance. (Figure by courtesy of A. Zisserman.)





gure 3. (*a*) An electrical plug, as seen by an ADEPT robot's rist-mounted camera. Its outline is captured by a 'dynamic ›ntour' tracker. Once captured, the contour is frozen and ıalysed for grip. (*b*) Seed grips shown here are all of 'type , grips that remain in equilibrium as the coefficient of iction approaches zero. They are parallel grips, in which ›e two fingers (depicted by rectangles) are placed at ›pposing points on the contour. Four such grips are shown ›re. The length of each rectangle depicts a measure of ability for one finger in a two-fingered grasp.






igure 4. (*a*) A screwdriver, as seen by the robot's camera, ith a 'dynamic contour' locked onto the outline of its andle. (*b*) There are four seed grasps of type 1 (see figure , the most stable of which are the natural grasps across the andle. The other, much less stable but still feasible, is ngitudinal.



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